Choosing the Discount Factor for Estimating Economic LGD

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Abstract

Banks must measure the loss arising from counterparty default in order to achieve Advanced-IRB compliance under the proposed Basel II minimum regulatory capital framework. Which discount rate to use on cash received post-default is a question that is the subject of considerable disagreement amongst practitioners and banking supervisors. We review alternative extant proposals and develop a new method for choosing an appropriate discount rate contingent upon the risk of the recovery cash flow. An example of how supervisory determined LGD discount rates could be set is demonstrated. Empirically, the required rate of return on defaulted corporate bonds is shown to be similar in magnitude to the yield on BB rated debt. For defaulted small and medium enterprise (SME) bank loans, the mean discount rate is found to be similar, on average, to the contract rate pertaining at the time of default.

1 Introduction

Banks that intend to measure their minimum regulatory capital under the Advanced-IRB option within the Basel II regulatory framework (BIS (2003)),

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must use an internal estimate of economic loss arising from counterparty default, termed the Loss Given Default, or 'LGD'. Because loan workout periods may extend over several years, it is necessary to discount post-default net cash flows to a common point in time; the most suitable being the event of default.[1] The question is what discount rate to apply. The answer is not trivial as it can affect the level of Pillar 1 capital proposed to be held by Advanced-IRB banks. Several alternative methods have been proposed by the industry and regulators indicating that little agreement exists.

2 A Survey of Methods

Under standard methods for evaluating bank solvency (Gupton, Finger, and Bhatia (1997)), the probability density of the value of the bank's loan portfolio is simulated over a given time-horizon. The probability density function includes variation in portfolio value from potential rating transitions, spread movements, and obligor default. The difference between the expected value of the portfolio and the maximum likely diminution in market value is termed economic capital and has formed the basis of internal VaR-based credit portfolio modelling for some years.

More recently, prompted by the growing arbitrage between regulatory and economic capital, bank supervisors have proposed a more risk-sensitive measurement of the minimum regulatory capital that is conceptually consistent with advanced internal bank economic capital models. Under BIS (2003) banks that meet minimum standards, may qualify to measure the LGD of their own loans, where LGD is defined in terms of the present value reduction in bank value expressed as a percentage of the exposure at default (EAD).[2]

The measurement of LGD is a standard problem in finance; the valuation of uncertain future cash flows. The only complication is that the cash flows relate to proceeds expected from post-default resolution or sale of obligor assets, net of recovery costs incurred by the bank, and not the original pre-default contractual promised payments. Let δ be a fixed spread over a risk-

[1] It follows that LGD is dependent upon the definition of default which is defined under BIS (2003) to include obligor bankruptcy, and other events such as debt restructure with economic loss, and payment arrears in excess of 90 days.

[2] The EAD represents the legal claim by the bank on the obligor for credit extended including principal and interest outstanding at the time of default.

free term rate, r_t, then the empirical LGD can be defined as:

$$LGD = 1 - EAD^{-1} \sum_{t=1}^{T} \frac{C_t}{(1 + r_t + \delta)^t} \quad (1)$$

where, EAD is the exposure at default, δ is a fixed risk premium, C_t is the net cash flow at time t inclusive of positive flows received under contract from the borrower, or through asset sales, and negative cash flows arising from internal and external costs. The mean of the historical sample is then taken as the expected LGD when estimating the loan portfolio's value probability density function.

Table 1 summarises a variety of approaches proposed by researchers, or recommended by regulatory bodies, for determining an LGD discount rate. The lack of agreement is surprising. There are basically three methods for estimation recovery rates. The first relies upon direct observation of defaulted debt prices in the market place. For example, Carty and Lieberman (1996) and Gupton and Stein (2002) compare the secondary price of defaulted bonds and bank loans one month after bankruptcy relative to par value. The method is a suitable measure of loss for investors who sell their debt immediately after default, but is not necessarily an accurate measure of loss if the debt is worked out by the bank to eventual resolution. A variation is provided by Carty, Hamilton, Keenan, Moss, Mulvaney, Marshella, and Subhas (1998) in which recovery is defined as the secondary market value of restructured assets, post emergence from bankruptcy proceedings, typically 18 months after default.

The second approach relies upon inference of recovery rates from non-defaulted bond prices. The method requires a model of credit risky debt valuation. For example, given the current credit spread and an estimate of asset volatility, Altman, Resti, and Sironi (2001) invert the implicit recovery rate from Merton (1974). The resulting recovery rate is obtained under the risk-neutral measure and includes a premium for the market price of risk due to correlation of the firm with the market. Bakshi, Madan, and Zhang (2001) infer plausible risk-neutral recovery rates from secondary prices using a reduced-form model. A difficulty with the inference method is that the expected recovery rate includes aggregate risk aversion that is not separately identified.

The third method, favoured by banks and banking supervisors, is to sample ex-post realised recoveries. Its advantages include: application to non-traded debt, a more direct measure of expected realisations under the physical measure, and ready verification of historical cash flow data obtained during the workout process. It therefore provides a richer set of information directly relevant to bank risk management and capital adequacy measurement. This method has been used by Edwards and Asarnow (1995) and Eales and Bosworth (1998) to estimate bank loan LGD and is commonly applied in practice, however, estimating the value of realised recoveries requires a discounted cash flow model of defaulted debt.

3 Post-Default Debt as an Asset Class

Clarity on the correct valuation model is provided by an empirical study by Guha (2003). Interestingly, the study finds that regardless of promised yield and remaining term-to-maturity, secondary market prices of equal ranking traded corporate bonds, converge to a common fractional value of par as the issuing firm approaches default. This suggests that the expected cash flows of issued debt converge. Prior to default, bonds are valued as the sum of discounted expected payments where the timing of default and amount of recovery is uncertain and varies across different bonds. Different yields to maturity reflect variations in cash flow structures and term. As default becomes imminent the instrument changes to represent a common claim on the restructured assets of the firm that may be viewed as equivalent to an asset worth the expected share of the post-bankruptcy pool of assets. The pre-default yield is no longer relevant to the valuation of post-default cash flows when the assets of the firm are to be legally passed to the creditors. The result is theoretically consistent with Merton (1974); for a given expected return on assets, credit spreads are predicted to increase with gearing. Should the firm default, the debtholders receive direct ownership in the firm's assets and expect to earn the rate of return of the firm's assets.

It is therefore reasonable to state that post-default cash flows represent a financial asset and therefore should be valued using a discount rate commensurate with the systematic risk associated with the asset class (a position similarly stated by Schuermann (2003)). Extending this view further, within the classic single-period CAPM framework, we can state that:

- losses incurred from a shortfall of realised assets relative to those contractually agreed should be modelled by a reduction in expected cash flows, where expectations are based on historical experience and are unbiased under the physical measure (a 'haircut');

- undiversifiable correlation between post-default cash flows and the market portfolio will give rise to a risk premium related to the Beta of the firm's unleveraged assets to the market;
- where collateral, unrelated to the firm's default risk rating, is taken as security, the risk of the recovery is related to the secondary market price of the collateral, and hence its market Beta, and not necessarily the firm's asset Beta;
- the LGD discount premium (δ) may equal zero if the lender holds full recourse to cash collateral, however, where a longer-term default-free asset is taken as security, a premium may still remain if the instrument's value is correlated to the market return (for example, a long-term Treasury bond);
- the discount rate should vary according the source of repayment and so several discount rates may be applicable when valuing a single firm's recovery.

We now have an appropriate framework to consider some alternative methods critically:

- **Contractual loan rate**. The rationale commonly cited is that this represents the opportunity cost of replacing the promised return on the defaulted loan.[3] The problem with this method is that it confuses pre and post default required returns in the event that payment comes from the liquidation of assets. As discussed above, default may result in a change in the nature of the financial claim creating a new instrument where the bank is a direct investor in the recoverable assets. Post default the required rate of return will vary depending upon the systematic risk of the new financial claim. An appropriate discount rate may therefore be less than the contractual rate which includes compensation for the expected reduction in cash flows relative to promised payments.

[3] A variation on this method is to apply a penalty rate that the lender has become entitled to as a consequence of default.

- **Lender's Cost of Equity**. The rationale is that shareholders cover the cost of recapitalising the bank's balance sheet. This method mistakenly replaces the systematic risk of the defaulted debt with the risk of the bank. The two are different investments and would result in the LGD rate varying with the bank's leverage and risk premium. Valuing an asset should be made from the perspective of a market clearing price considering what a buyer will pay and not solely by what the seller wishes to recoup to repair their balance sheet.

- **Risk-Free Rate**.Recent research has demonstrated that recovery rates are systematically related to economic conditions (Frye (2000), Altman, Resti, and Sironi (2001), and Gupton and Stein (2002)). Defaulted debt is therefore likely to carry an expected rate of return strictly greater than the risk free rate, unless the bank expects payment from liquidation of cash collateral.

- **Ex-post Defaulted Bond Returns**. Araten (2004) reports that JP Morgan Chase use a discount rate of 15% to reflect the average yield over time required by the buyers of distressed debt with reference made to the historical performance of the Moody's Bankrupt Corporate Bond Index(Hamilton and Berthault (2000)). Over 19 years the index has returned 17.4%, but the historical return has been extremely volatile and is dependent upon the period of history chosen. For the decade to 1991, the index gained little, with most of the 17.4% earned during 1992 to 1996 in which the index earned 147%. For this reason, a more meaningful measure of expected, as opposed to realised return, can be achieved by examining the systematic risk of the index. Hamilton and Berthault (2000)) report that the market Beta, measured relative to the S&P 500, is a modest 0.2, supported by a similar study by Altman and Jha (2003).

Returning to Table 1, it can be seen that the FSA proposal is a correct statement of the economic problem with direct relevance in the situation where the loan has transformed to an investment in the firm as per the theoretical model of Merton (1974).

However, it is frequently observed in commercial banking that resolution does not arise from transfer of ownership of the underlying firm. If rather than sale of assets, recovery arises from entering a new contract to pay, or from enforcing an existing contract, then the recovery cash flows remain subject to default risk and should be discounted at a rate commensurate with the bank's required return on the relevant contract. The distinction between asset sales and post-default contracts is important as this effectively ensures that the discount rate on renegotiated loans and technical defaults is at least as high as the original contractual rate.

There are several ways in which recovery under contract arises. Firstly, the bank may enter distressed negotiations to receive a lower than originally agreed payment. Secondly, the borrower may dip into technical default, for example by temporarily exceeding the 90 days in arrears Basel II definition of default, but thereafter correct the arrears under the original contract. Thirdly, the borrower may refinance with another lender and repay the debt in full. In these situations the lender has received cash under a financial contract that carries default risk and not asset sale risk. The discount rate applied to these recovery cash flows should reflect the internally required rate of return necessary to compensate for the default risk of the obligor standing behind the post-default contract. For payments made by the borrower under the original contract, the required return should reflect the heightened risk evidenced by the default, but where a third-party guarantee is held, cash received under the guarantee should be discounted at a required rate commensurate with the default risk of the guarantor.[4]

The OCC (2003) proposal is somewhat similar, but the suggestion of making the discount rate equivalent to the lowest-quality grade loan rate may underestimate the true default risk of the post-default contract, and certainly fails to acknowledge the reduced risk associated with payments from low-risk guarantors. The other methods, with the exception of Eales and Bosworth (1998), are largely based on the original contractual rate of return which is not supported theoretically.

4 A Risk-Based LGD Discount Rate Method

An Advanced-IRB bank is likely to have in place the capability of recording the amount, timing, and source of cash received on defaulted loans.[5] The

[4]For example, payments received under a bank guarantee, or bought credit swap, carry the default risk of the bank, or swap counterparty, and not the borrower.

[5]BIS (2003, para 432) states that 'LGD estimates must be grounded in historical recovery rates and, when applicable, must not solely be based on the collateral's estimated market value.' The ability to model expected LGD is therefore dependent upon identifying

latter is particularly relevant as it determines the correct choice of discount method. We now turn attention to the situation where cash is not received as agreed with a defaultable promise to pay, so the fair rate of return expected from sale of assets must be performed.

The BIS (2003) capital formulae are developed in Vasicek (2002) and Gordy (2000) from the Merton (1974) structural modelling framework. Changes in the value of borrower i assets, V_i are assumed to be correlated to an independent standard normally distributed systematic factor, X, and a firm-specific factor, Z_i, where the correlation is denoted by $\rho_{i,x}$

$$V_i = \rho_{i,x} X + \sqrt{1 - \rho_{i,x}^2} Z_i. \tag{2}$$

It follows that the pairwise asset-correlation between firm i and j is $\rho_{i,j} = \rho_{i,x}\rho_{j,x}$.

By assuming a single systematic asset factor and infinite granularity, Vasicek (2002) demonstrates that a portfolio invariant solution for economic capital is achieved under the assumption that LGD is not systematically correlated with firm asset returns. For specific asset classes BIS (2003) defines the common correlation factors, $\rho_{i,x}^2$, to be applied equally across all banks regardless of bank portfolio exposure to industry or geography. Given this single-factor, portfolio invariant world, it is natural to make the economic interpretation that $\rho_{i,x}^2 = \rho_{i,m}^2 = R_i$, where m denotes the market portfolio and R_i is the denomination of the asset-correlation in the Advanced-IRB capital formulae. The rate of return expected on the firm's assets is then related to the market by the firm's asset Beta

$$\beta_{i,m} = \frac{\sigma_i \rho_{i,m}}{\sigma_m} = \frac{\sigma_i \sqrt{R_i}}{\sigma_m}. \tag{3}$$

It follows that the expected rate of return on defaulted debt is the expected return on the assets of the firm where the excess return is

$$\begin{aligned} \delta_i &= \beta_{i,m} MRP \\ &= \frac{\sigma_i \sqrt{R_i}}{\sigma_m} MRP \end{aligned} \tag{4}$$

where MRP denotes the market risk premium which is assumed to be con-

net proceeds from specific collateral sales.

stant.

In summary, default correlation in the Basel II framework is based on an underlying assumption of asset correlation to a common systematic factor that can reasonably be interpreted as a market portfolio return. For internal consistency within the Basel II modelling framework, it is therefore desirable that the same asset return correlation assumption be used for determining the LGD discount rate.

A special case exists if the loan is secured by a specific asset with risk-return characteristics different from the general assets of the firm, then $\rho_{i,m}$ may be considered the collateral asset's systematic risk rather than the general assets of the firm; examples being cash collateral or business loans secured by third party guarantees over unrelated property. In these situations we have a potential difference between the asset correlation driving default correlation and the asset correlation driving the equilibrium LGD discount rate. It is not difficult then to see that for more complex lending relationships, the cash flows received from realisations of different asset classes may be discounted at different rates if their systematic risk is different. Failing to distinguish between say secured and unsecured asset Betas (as in Frye (2000)), may overstate the impact of PD-LGD correlation on bank capital to the extent that the bank relies on collateral independent of the obligor's default risk.[6]Figure 1 shows a decision method for selecting an appropriate LGD discount rate by first establishing whether the recovery cash flow is related to payment by contract, and then to distinguish between assets with different implied systematic risk within the Basel II framework.

[6]Concern over PD-LGD correlation impact on the estimation of capital has potentially influenced supervisors to adopt a conservative worst-in-the-cycle calibration of LGD.Frye (2003) states...

> Ideally, credit models would incorporate this [correlation] behavior. Until the appropriate changes are made, a short-term alternative is to redefine LGD, for capital purposes. Rather than multiplying by historically observed LGD, practitioners could multiply by a function of the historical level that projects a level of LGD appropriate for an economic downturn.

5 Estimates of Corporate Debt LGD Risk Premium

Frye (2000) extends Vasicek (2002) and Gordy (2000) to let the recovery rate be systematically related to the common factor. Thus, aggregate level default rates and the expected LGD are correlated via their common loading on the credit cycle represented by variation in the systematic market factor. For borrower i, the LGD is a function of a scalar,μ_i, firm asset volatility, σ_i, asset correlation to the market,$q_{i,m}$, to a single market factor, M, and to an idiosyncratic factor, Z. The factors M and Z_i are independent standard normal variables.

$$1 - \frac{LGD_i}{EAD_i} = \mu_i + \sigma_i q_{i,m} M + \sigma_i \sqrt{1 - q_{i,m}^2} Z_i \tag{5}$$

Frye (2000) undertakes an empirical estimation of (5) allowing the asset correlation determining joint default, $p_{i,m}$, to be different from the asset correlation determining recovery, $q_{i,m}$. He estimates, on average, $p_{i,m} = 0.23$ and $q_{i,m} = 0.17$ using corporate bond losses reported by Moody's over the period 1983-1997. As hypothesised by Merton (1974), the two parameters are very similar confirming the reasonableness of assuming $\rho_{i,m} = p_{i,m} = q_{i,m}$.

Table 2 illustrates how the LGD discount rate varies with $\rho_{i,m}$ according to (4). When systematic risk is zero, the discount rate equals the risk-free rate. As the firm's assets become more correlated to the market the discount rate increases and the premium extends over a range of values from 0 to 373 bps. For the average correlation estimated by Frye (2000) of $\rho_{i,m} = 0.17$, the discount premium is 181 bps.

We test Frye's average asset correlation estimate by direct observation of defaulted debt returns sourced from the Altman-NYU Salomon Center Index of Defaulted Public Bonds (Altman and Jha (2003)). The correlation of return with the S&P 500 (as a proxy for the market) over the period 1987-2002 is a similar 0.20 implying an LGD risk premium of 216bps. This is a similar to the average credit spread on non-defaulted BB rated corporate debt reported by Davydenko and Strebulaev (2002, Table 2) over the period 1994-1999.

A further direct measurement of the historical defaulted debt Beta to the S&P 500 was performed using monthly secondary market bid quotes (April 2002 to August 2003) on defaulted US debt with data obtained from a secondary market dealer (refer Table 3). The historical Beta is significant with an estimate of 0.37, which from the assumptions used in Table 2, implies a correlation of 0.21 and an LGD risk premium of 224bps. Taking the average of the three above estimates gives an LGD risk premium of 207bps for defaulted US corporates.

6 Regulatory Guidance

Regulatory guidance for the choice of LGD discount rates should ideally be: a) well founded in finance theory for longevity and robustness of the solution, b) consistent with the Basel II capital model assumptions to avoid unintended results, and c) be easily and consistently applied across supervised banks, but with sufficient richness to encourage banks to adopt and enhance it for internal purposes.

Table 4, combined with the decision logic detailed in Figure 1, illustrates how a risk-based model could be used to establish common LGD discount factors across banks; an essential element of ensuring comparable risk measurement and regulatory treatment. In Table 4 the LGD discount factors are varied by regulatory asset class on the basis that these assets have different prescribed asset correlations within the BIS (2003) capital functions. All assumptions other than the current risk free rate could be set by agreement by local supervisors to ensure initial consistency between banks, however, for internal economic capital measurement, banks could use their own estimates.

Note that we have used a single value of $\beta_{i,m}$ for the asset classes without regard to probability of default. Under a more advanced internal approach, it is preferable to use the borrowing firm's specific asset correlation estimate when calculating LGD discount rates. Crouhy, Galai, and Mark (2000) describe a widely used method of estimating corporate default correlation as a function of company size (turnover), geographic location, and industry membership. Under the existing BIS (2003) capital functions, pair-wise asset correlation (R_i) is assumed to increase as default risk decreases, thus, the more complex factor description of firm asset correlation is collapsed into a single-factor representation that is negatively related to default risk by an associated correlation with obligor size. If we were to use the same default rate based BIS (2003) asset correlations that are embedded in the Pillar 1 capital formulae, then our estimate of asset correlation would be biased inadvertently downwards due to rating migration preceding default. It is therefore reasonable to control for default risk by using an average correlation factor per regulatory asset class as shown in Table 4.[7] Table 4 shows a disparity with the direct measurement of correlation obtained from Frye's maximum likelihood estimation, the regression of the Altman-NYU defaulted bond index, and the regression of secondary market defaulted debt bid quotes. At $R_i = \rho_{i,m}^2 = 0.20$ the implied discount premium on corporate loans is 480bps compared with the average 207bps reported above. Lowering the assumed asset volatility in Table 4 from 0.32 to 0.20 decreases the LGD discount premium to 300bps suggesting that the proposed regulatory LGD discount model can be further calibrated successfully.

The treatment of retail mortgage correlation in BIS (2003) is discussed further in Calem and Follain (2003) in which the choice of $R_i = 0.15$ is defended empirically for 30 year fixed-rate US mortgage loans. For countries, such as Australia, where variable rate mortgages are the dominate form of residential financing, the value risk is considerably less. Therefore, for illustrative purposes, the LGD discount rate assuming $R_i = 0.05$ is also shown.

7 An Empirical Study of Bank Loans

In this section, we apply the discount rate method described in Figure 1 and Table 4 to actual bank default data. Post-default cash flows are obtained from internal ANZ Banking Group empirical study of defaulted loans from February 1998 to September 2003. This period has been a benign part of the credit cycle in Australia and the estimates of LGD reported are likely to below long-term unconditional averages. The sample comprising 209 SME borrowers with annual turnover less than € 50m. Within the sample we further segment into 35 large SME borrowers with turnover greater than AUD 10m, and 174 smaller SME borrowers with turnover less than AUD 10m.[8] Measures of R_i for the two segments is obtained by applying a representative

[7] The asset correlation in BIS (2003) varies inversely with PD as follows: corporate, sovereigns and banks range between 0.12 to 0.24; high-volatility commercial real estate ranges from 0.12 to 0.30; qualifying revolving retail exposures range between 0.02 and 0.11, other retail exposures range between 0.02 and 0.17; and, residential mortgages are fixed at 0.15. The SME function overlays a sales turnover based adjustment to the corporate function.

[8] The segmentation is based on differential levels of relationship management activity with the larger segment subject to more active management of limits that lessens the occurrence of minor excess arrears based defaults.

turnover value and default rate into the Basel II correlation function BIS (2003, para 242). The resulting values of R_i are 0.20 for the large segment and 0.15 for the small segment. The LGD discount premiums from table 4 are applied with the large SME segment represented by the corporate asset class. For simplicity, the LGD discount premium is added to a current one-year risk-free rate of 5.56%, although in practice the term structure prevalent on the day of default should be used as the risk-free rate. The average exposure at default for large and small SME borrowers was AUD 6.8m and AUD 0.75m respectively. Table 5 shows that the predominant source of recovery for the large segment was firm asset liquidation (54.6%), followed by liquidation of commercial real estate (24.6%), then cash recovered under the original contract (13.3%). A traditional structural modelling would have understated the average discount rate by not including contract payments. For the small borrower segment a significantly different pattern is evident with the highest source of recovery coming from payments under the original contract, including refinance (57.8%), followed by liquidation of commercial (16.5%) property, then residential property (13.9%). Liquidation of firm assets is infrequent at only 9.1%. The mean discount rate for the larger segment is 9.61% compared to the mean original contractual rate of 9.59%. Whilst similar on average, the proposed LGD discount method generates a lower risk premium on corporate assets held for resale but a higher rate on renegotiated debt. For smaller borrowers, the proposed LGD discount method results in a higher mean discount rate at 10.21% compared to the mean original contractual rate of 9.37%.

Table 6 reports the mean LGD expressed as a percentage of EAD and Figure 2 shows a plot of the empirical LGD frequency distributions. Immediately noticeable is the much lower loss associated with small SME borrowers which is due to a higher proportion of cash recovered under the original debt contract from arrears correction and refinance.

8 Conclusion

By applying a simple CAPM valuation framework to defaulted debt it becomes clear that the appropriate method for the LGD discount rate is that proposed by FSA (2003). However, the OCC (2003) proposal is similar to the suggested method for valuing post-default cash flows recovered under contract. However, the OCC (2003) proposal fails to recognise that the discount rate should vary with the obligor's credit risk which is unlikely to be equivalent to the bank's required credit spread for the lowest-quality grade on new origination.

A method for choosing an appropriate discount rate based upon on the source of repayment is therefore developed, and a tractable method for estimating the liquidation discount rate is explored that requires estimation of asset correlation to the market. This neatly fits within the single-factor portfolio modelling framework of BIS (2003) as developed in Gordy (2000). Further calibration of asset correlation and asset volatility assumptions appears warranted to more closely match direct measurement of historical returns.

From three different sources of data on defaulted US corporate debt returns, an average LGD discount rate of 207bps was estimated thereby refuting the much higher suggestions made by Eales and Bosworth (1998) and Araten (2004). The proposed method of determining the discount rate was applied to actual SME bank defaults, and the mean LGD was found to be 37.2% and 11.8% across large and small SME borrowers respectively. The results are very close on average to the contractual rates that prevailed on the loans at the time of default.

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Table 1: Alternative Discount Rate Methods

Source	Method
Edwards and Asarnow (1995)	Contractual loan rate including penalty
Eales and Bosworth (1998)	Lender's cost of equity
Friedman and Sandow (2003)	Coupon rate
Gupton and Stein (2002)	Market value one month post-default
Carty, Hamilton, Keenan, Moss, Mulvaney, Marshella, and Subhas (1998)	Contractual loan rate
Araten (2004)	15% flat rate justified by ex post realised returns on the Moody's Bankruptcy Bond Index (Hamilton and Berthault (2000))
OCC, the Board of Governors, FDIC, and OTS (2003)	The discount rate must be no less than the contract interest rate on new originations of a type similar to the transaction in question, for the lowest-quality grade in which a bank originates such transactions. Where possible, the rate should reflect the fixed rate on newly originated exposures with term corresponding to the average resolution period of defaulting assets (paragraph 134)
FSA (2003)	Firms should use the same rate as that used for an asset of similar risk. They should not use the risk free rate or the firms hurdle rate (unless the firm only invests in risky assets such as defaulted debt instruments) (page 68, Annex 3)
IAS 39 (2003)	Effective original loan rate (the rate that exactly discounts expected future cash payments or receipts through the expected life of the financial instrument)

Table 2: LGD Discount Factor by Correlation

This table illustrates how the LGD discount factor varies for typical values of asset correlation, $\rho_{i,m}$. Asset volatility, σ_i, is sourced from Frye (2000, Table A) and market volatility, σ_m, is the historical annual S&P 500 return volatility from 1987-2002. The annual risk-free rate, r_t, is assumed to be 0.05, and the Market Risk Premium, MRP, is assumed to be 0.06. Shown separately are the discount rates for Frye's (2000) average recovery asset correlation of 0.17 obtained by maximum likelihood from bond loss time-series distributions, and a correlation of 0.203 obtained by direct measurement of monthly defaulted bond returns with the S&P 500 by Altman and Jha (2003).

$\rho_{i,m}$	σ_i	σ_m	$\beta_{i,m}$	r_t	MRP	LGD Discount	δ_i (bps)
0.350	0.32	0.18	0.62	0.05	0.06	0.0873	373
0.300	0.32	0.18	0.53	0.05	0.06	0.0820	320
0.250	0.32	0.18	0.44	0.05	0.06	0.0767	267
0.200	0.32	0.18	0.36	0.05	0.06	0.0713	213
0.150	0.32	0.18	0.27	0.05	0.06	0.0660	160
0.100	0.32	0.18	0.18	0.05	0.06	0.0607	107
0.050	0.32	0.18	0.09	0.05	0.06	0.0553	53
0	0.32	0.18	0.00	0.05	0.06	0.0500	0
Frye (2000)							
0.170	0.32	0.18	0.30	0.05	0.06	0.0681	181
NYU Bond Index							
0.203	0.32	0.18	0.36	0.05	0.06	0.0716	216

Table 3: Market Beta on Defaulted Debt Bid Quotes

This table reports the time series of excess returns on a sample of 90 US defaulted corporate bonds. Bid quotes were provided by a leading secondary market trader. The empirical regression model is:

$$R_{i,t} - r_t = \alpha_i + \beta_{i,m}(R_{m,t} - r_t) + \epsilon_t \quad (6)$$

-0.004	0.371
(0.318)	(1.773)

T-stats are reported in parentheses with $\beta_{i,m}$ significant at the 10% level.

Month:Year	Defaulted Debt Return $R_{i,t}$	S&P 500 Index Return $R_{m,t}$	10 yr Treasury r_t	Excess Defaulted $R_{i,t} - r_t$	Excess Market $R_{m,t} - r_t$
04:2002	1.26%	1.89%	0.43%	0.83%	1.45%
05:2002	3.54%	-0.53%	0.43%	3.11%	-0.96%
06:2002	-1.97%	-7.11%	0.41%	-2.38%	-7.52%
07:2002	-5.19%	-12.36%	0.39%	-5.57%	-12.75%
08:2002	-11.25%	-0.60%	0.36%	-11.61%	-0.95%
09:2002	-2.99%	-3.13%	0.32%	-3.32%	-3.46%
10:2002	-6.55%	-3.74%	0.33%	-6.87%	-4.06%
11:2002	1.98%	5.70%	0.34%	1.64%	5.36%
12:2002	6.80%	0.58%	0.34%	6.47%	0.24%
01:2003	4.46%	-1.08%	0.34%	4.12%	-1.42%
02:2003	1.63%	-6.16%	0.33%	1.30%	-6.49%
04:2003	-1.20%	-0.21%	0.33%	-1.53%	-0.54%
05:2003	-1.13%	4.64%	0.30%	-1.43%	4.34%
06:2003	4.30%	6.93%	0.28%	4.02%	6.65%
07:2003	1.95%	6.72%	0.33%	1.62%	6.39%
08:2003	0.26%	2.85%	0.37%	-0.11%	2.48%

Table 4: Example of Regulatory LGD Discount Rates

This table presents an example of a simple regulatory LGD discount rate. Asset classes conform to BIS (2003) capital function definitions. Input assumptions include the pair-wise asset correlation, R_i, asset volatility, σ_i, market volatility, σ_m, and the Market Price of Risk, MPR. Calculating as per (3) and rounding gives an 'agreed' supervisory market Beta and LGD risk premium to be added as a margin to the term structure of risk-free rates prevailing at the date of default. Values of R_i are chosen to represent values in the BIS (2003) capital functions for a typical bank portfolio. An exception is residential mortgages, where R_i is reduced from the current BIS (2003) proposal of 0.15 to 0.05 to illustrate lower risk variable rate mortgages. HVCRE refers to high volatility commercial real estate.

Asset Class	R_i	σ_i	σ_m	$\beta_{i,m}$	'Agreed' $\beta_{i,m}$	MPR	LGD Premium δ_i (bps)
HVCRE	0.3	0.32	0.18	0.97	1.00	0.06	600
Corporate	0.2	0.32	0.18	0.80	0.80	0.06	480
SME	0.15	0.32	0.18	0.69	0.70	0.06	420
Revolving Retail	0.1	0.32	0.18	0.56	0.55	0.06	330
Other Retail	0.1	0.32	0.18	0.56	0.55	0.06	330
Mort. (US 30yr Fixed)	0.15	0.32	0.18	0.69	0.70	0.06	420
Mort. (Low Risk)	0.05	0.32	0.18	0.40	0.40	0.06	240

Table 5: Empirical SME Bank Loan LGD Discount Rates, Feb 1998 - Sep 2003

This table shows the asset class discount rates obtained from a sample of post default cash flows surveyed by ANZ Banking Group. The sample period is a benign part of the Australian credit cycle and the LGD rates shown are likely to be below the unconditional long-run average. Net proceeds recovered from uncollateralised guarantee payments are discounted at the internal required new lending rate on an unsecured exposure for the risk of the guarantor; in this sample the worst non-default PD rating. Net proceeds recovered under contract are discounted at the required internal new lending rate for the worst non-default PD rating with the sample average level of collateral security. Liquidation discount rates are the sum of the current one-year risk-free rate (5.56%) and the LGD premia from Table 4 with large SME the same as corporate. Contract rates refer to the mean yields on the defaulted loans at the time of default.

Source of Net Cash	LGD Discount Rate (%)	Contract Rates (%)	LGD Discount Premium (bps)	Contract Spread (bps)	Sample Contrib. (%)
Large: Sales>AUD10m					
Guarantee Payment	15.46	7.48	990	192	0.17
Liquidation-SME Large	10.36	12.07	480	651	54.61
Liquidation-HVCRE	11.56	10.42	600	486	24.62
Liquidation-Res. Mortg.	7.96	11.07	240	551	5.19
Original Contract	11.56	10.34	600	478	13.26
Re-negotiated Contract	11.56	10.50	600	494	2.15
Total	10.72	11.34	516	578	100.0
Small: Sales<AUD10m					
Guarantee Payment	15.46	10.83	990	527	2.19
Liquidation-Cash	5.56	10.84	0	528	0.09
Liquidation-HVCRE	11.56	11.48	600	592	16.49
Liquidation-Res. Mortg.	7.96	10.14	240	458	13.89
Liquidation-SME Small	9.76	11.10	420	554	9.10
Original Contract	11.56	10.48	600	492	57.83
Re-negotiated Contract	11.56	9.53	600	397	0.41
Total	10.98	10.66	542	510	100.0

Table 6: Descriptive LGD Statistics for SME, Feb 1998 - Sep 2003

This table shows descriptive statistics of the LGD distribution after application of discount rates reported in Table 5. The sample uses actual cash flows obtained from a survey of defaulted customer accounts for the period Feb 1998 to Sep 2003. The sample period is a benign part of the Australian credit cycle and the LGD rates shown are likely to be below the unconditional long-run average.

	Sales>AUD10m		*Sales<AUD10m*	
	Mean Economic (%)	Mean Contract (%)	Mean Economic (%)	Mean Contract (%)
Mean	37.19	37.47	11.78	11.76
Median	31.45	29.31	2.00	1.36
Minimum	0	0.14	0	0
Maximum	101.48	101.43	102.28	102.09
Std.Dev.	32.72	32.70	24.11	24.21

Figure 1: Process for Selecting an Economic Discount Rate using Risk of Source of Repayment

This figure illustrates a risk-based method for choosing an appropriate discount rate within the Basel II framework. Net cash received post-default is given a discount rate premium, above the maturity matched risk-free rate prevailing at the time of default, contingent upon the source of the cash. Asset classes are chosen in accordance with the BIS (2003) capital functions reflecting different systematic asset risk. In a single workout, several discount premia may be applicable.

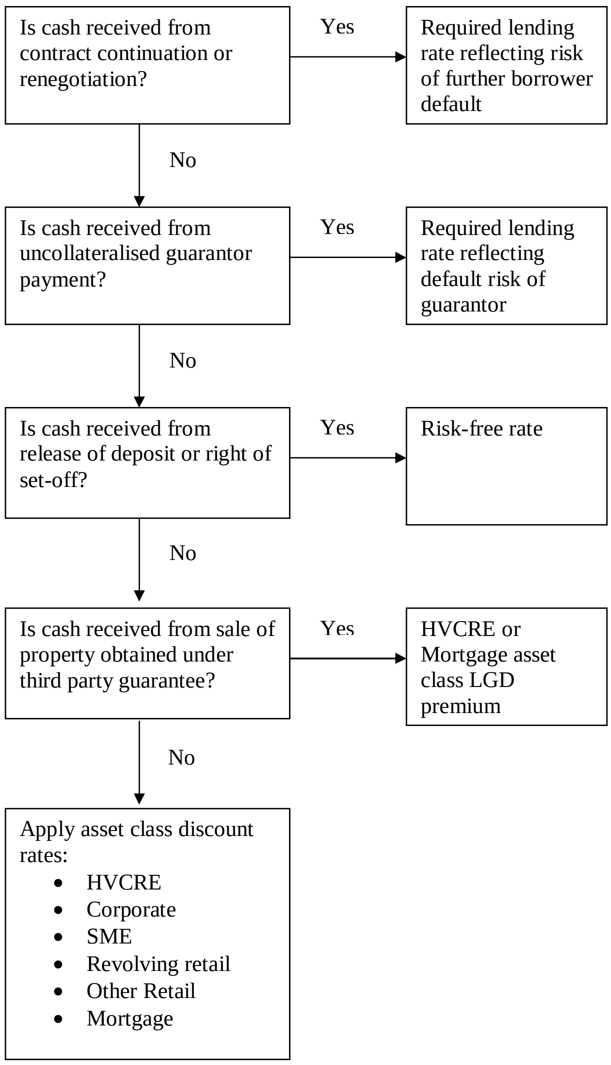

Figure 2: Empirical Bank Loan LGD Frequency Distribution

Plotted are the empirical frequency distributions of LGD expressed as a fraction of EAD for a sample of 35 Corporate and 174 SME borrowers. Refer Table 6.